

Precision measurement of charge number with optomechanically induced transparency

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We propose a potentially practical scheme to precisely measure the charge numbers of small charged objects by optomechanical systems using optomechanically induced transparency (OMIT). In contrast to the conventional measurements based on the noise backaction on the optomechanical systems, our scheme makes use of the small deformation of the mechanical resonator sensitive to the charge number of the nearby charged object, which could achieve the detection of a single charge. The relationship between the charge number and the window width of the OMIT is investigated and the feasibility of the scheme is justified by numerical simulation using currently available experimental values.

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I. INTRODUCTION

Precision measurement is one of the essential tasks in the study of modern physics. The (micro- or nano-) mechanical resonators (MRs) hold the promise for realizing precision measurements due to the possibility of presenting both classical and quantum properties [1, 2]. For the measurements approaching the quantum limit [3], we have to cool the MRs to their ground states and show obvious quantum behavior. Up to now, a great diversity of methods for MR cooling have been proposed in optomechanical or/and electromechanical systems, such as the feedback cooling [4–6], the backaction sideband cooling [7–9], the bang-bang cooling [10], the electromagnetically induced transparency cooling [11], the measurement-based cooling [12], and the thermal light cooling [13], where some of them have been achieved experimentally [14–19].

The precision measurement based on MRs can be classified by two kinds of systems, i.e., the optomechanical and electromechanical systems. We focus in the present work on the optomechanical system, in which the precision measurements are usually carried out by the correlations between the output spectra and the measured quantities, based on the reflected noise [3]. For example, the precision measurement of the MR's displacement with a factor of only 5 times higher than the standard quantum limit has been observed in the detection of the optical output spectrum [20], and a recent experiment about the displacement measurement of MR beyond the standard quantum limit has also been reported [21].

The optomechanically induced transparency (OMIT)

is a kind of induced transparency caused by radiation pressure to couple the light to the MR modes [23]. Recently, the OMIT in the optomechanical system has been theoretically predicted [22, 23] and also observed experimentally [23–26]. However, the application of the OMIT is still absent. As far as we know, the only one application is single photon routers in which the OMIT effect are used to control a probe field in a single photon Fock state [27].

The aim of the present work is to detect the charge number in a small charged body via OMIT in an optomechanical system. Since the window width of the OMIT depends on the mean photon number as well as the small deformation of the MR [22, 23], if we consider the additional Coulomb interaction between a charged MR and a charged body, which modifies both the steady-state position of the MR and the mean photon number in the cavity, the Coulomb force can be reflected by the modified window width of the OMIT. As a result, the charge number in the charged object is possibly detected by the OMIT.

Our study shows that the window width in some special regions of the OMIT varies with the charge number in a sensitive way, which makes it possible for a precision measurement of the charge number. The relevant ideas about the ultra-sensitive measurements in optomechanical systems, e.g., the cavity optomechanical magnetometer [28] and the displacement measurement [20], are based on the quantum noise backaction, in which the noise backaction can be suppressed by increasing the light intensity [3]. Different from these schemes, in our case, the output intensity (of the probe field) is monitored via OMIT which is a collective effect. Hence the noise backaction can be ignored. Moreover, the conventional MR electrometers are based on vibrating reed electrometers [29]. They are formed with moving and fixed electrodes, and can measure the Coulomb forces with variable ca-

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pacitors. Limited by the extremely sensitive in electricity, e.g., an extremely sensitive of current is 0.12aA [30], they are hard to measure the charge density of the small object in very small size ($< 6\text{nm}$ [31]). Conversely, since optical measurements take higher sensitive than the electrical ones, our scheme works well even for detecting single charge number in small objects with less size. Furthermore, our scheme makes use of the unique feature of optomechanical measurements which are considered to have higher sensitivity than the electromechanical measurements [32].

The paper is structured as follows: We present the model and Hamiltonian of the system in the next section, and study the output field for the OMIT in Sec. III. The relationship between the charge number and the output field as well as the feasibility of our scheme is described in Sec. IV. The last section is for a brief conclusion.

II. MODEL AND HAMILTONIAN

The model we consider is shown in Fig. 1, where a high-quality cavity consists of a fixed mirror and a movable one, i.e., a MR. Besides the radiation pressure force coupling the MR to the cavity mode, the charged MR is subject to the Coulomb force due to the charged body nearby. Such a system can be described as,

$$H_1 = \hbar\omega_c c^\dagger c + \left(\frac{p^2}{2m} + \frac{m\omega_m^2}{2}q^2\right) - \chi qc^\dagger c - \frac{n|e|Q_{MR}}{4\pi\epsilon(r_0 - q)} + i\hbar[(\varepsilon_l e^{-i\omega_l t} + \varepsilon_p e^{-i\omega_p t})c^\dagger - \text{H.c.}] \quad (1)$$

The first term is for the single-mode cavity of eigen-frequency ω_c with the bosonic annihilation operator c . The second term describes the vibration of the MR where q and p are, respectively, the position and momentum operators of the MR with the eigen-frequency ω_m and the effective mass m . The third term is for the radiation pressure coupling between the cavity field and the MR, where $\chi = \hbar\omega_c/L$ is the coupling strength with L being the cavity length. The forth term presents the interaction of the charged MR with the charged body via a Coulomb potential $V_c = \frac{-n|e|Q_{MR}}{4\pi\epsilon(r_0 - q)}$, where Q_{MR} is the positive charge on the MR, $-n|e|$ is for n negative charges of the charged body to be detected, and r_0 is the distance between the equilibrium positions of MR center of mass and the charged body in the absence of the radiation pressure and the Coulomb force. In our case with the attractive force, the Coulomb force on the MR takes the same direction as the radiation pressure force on the MR. The last term in Eq. (1) describes two optical drives to the cavity from the fixed mirror: One is the strong pumping field with frequency ω_l and the other is the weak probe field with frequency ω_p , and ε_l and ε_p are the corresponding driving strengths, respectively.

In the case of $q \ll r_0$, the Coulomb interaction can

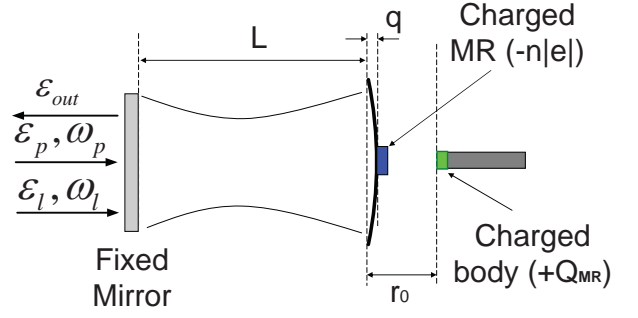


FIG. 1: Schematic diagram of the system. An optomechanical cavity with the length L is driven by two light fields. One is the pumping field ε_l with frequency ω_l and another is the probe field ε_p with frequency ω_p . The output field is represented by ε_{out} . r_0 is the distance between the charged body and charged MR in the absence of the radiation pressure and the Coulomb force. Under the action of the radiation pressure and the Coulomb force, the MR takes a position q . Here, the charges on the charged body and charged MR are $-n|e|$ and Q_{MR} , respectively.

be rewritten as $V_c \simeq -\frac{n|e|Q_{MR}}{4\pi\epsilon r_0}(1 + \frac{q}{r_0})$. Omitting the constant term, we have the Hamiltonian in the frame rotating with the driving frequency ω_l ,

$$H_2 = \hbar\Delta_c c^\dagger c + \frac{1}{2m}(p^2 + m^2\omega_m^2 q^2) - \chi qc^\dagger c - n\eta q + i\hbar[(\varepsilon_l + \varepsilon_p e^{-i\delta t})c^\dagger - \text{H.c.}] \quad (2)$$

with $\Delta_c = \omega_c - \omega_l$, $\eta = |e|Q_{MR}/(4\pi\epsilon r_0^2)$, and $\delta = \omega_p - \omega_l$. Here both ε_l and ε_p are complex.

III. MEAN-VALUE EQUATIONS AND QUADRATURES OF THE OUTPUT FIELD

By analyzing the mean response of the OMIT, we may neglect the quantum fluctuation of the system [22] and consider the Langevin equations [33]. In our case, the mean-value equations of the system are written as,

$$\begin{aligned} \left\langle \frac{dq}{dt} \right\rangle &= \frac{\langle p \rangle}{m}, \\ \left\langle \frac{dp}{dt} \right\rangle &= -m\omega_m^2 \langle q \rangle + n\eta + \chi \langle c^\dagger c \rangle - \gamma_m \langle p \rangle, \\ \left\langle \frac{dc}{dt} \right\rangle &= [\kappa - i(\Delta_c - \frac{\chi}{\hbar} \langle q \rangle)] \langle c \rangle + \varepsilon_l + \varepsilon_p e^{-i\delta t}, \end{aligned} \quad (3)$$

where κ and γ_m are introduced as the decay rates of the cavity and the MR, respectively. Eq. (3) can be solved under the condition that the pumping field is much stronger than the probe one. Since Eq. (3) is a nonlinear equation, the steady-state response in the frequency domain is composed of many frequency components. As a result, we suppose the steady-state solutions to Eq. (3)

taking the form of

$$\begin{aligned}\langle q \rangle &= p_s + p_+ \varepsilon_p e^{-i\delta t} + p_- \varepsilon_p^* e^{i\delta t}, \\ \langle p \rangle &= q_s + q_+ \varepsilon_p e^{-i\delta t} + q_- \varepsilon_p^* e^{i\delta t}, \\ \langle c \rangle &= c_s + c_+ \varepsilon_p e^{-i\delta t} + c_- \varepsilon_p^* e^{i\delta t},\end{aligned}\quad (4)$$

where each solution contains three items O_s , O_+ and O_- (with $O = p, q, c$), corresponding to the responses at the original frequencies ω_l , ω_p , and $2\omega_l - \omega_p$, respectively [33]. Since $O_s \gg O_{\pm}$, Eq. (4) can be solved by treating O_{\pm} as perturbation. After combining Eq. (4) with Eq. (3), and ignoring the second-order small terms, we obtain the steady-state mean-values of the system by resorting the prefactors in terms of the exponentials $e^{\pm i\delta t}$,

$$\begin{aligned}p_s &= 0, & q_s &= \frac{\chi |c_s|^2 + n\eta}{m\omega_m^2}, \\ c_s &= \frac{\varepsilon_l}{\kappa + i\Delta}, & |c_s|^2 &= \frac{|\varepsilon_l|^2}{\kappa^2 + \Delta^2},\end{aligned}\quad (5)$$

with $\Delta = \Delta_c - \frac{\chi}{\hbar} q_s$, and the solution to c_+ [22] is

$$c_+ = \frac{(\delta^2 - \omega_m^2 + i\gamma_m \delta)[\kappa - i(\Delta + \delta)] - 2i\omega_m \beta}{[\Delta^2 + (\kappa - i\delta)^2](\delta^2 - \omega_m^2 + i\delta\gamma_m) + 4\Delta\omega_m \beta}, \quad (6)$$

with $\beta = \frac{\chi^2 |c_s|^2}{2m\hbar\omega_m}$.

Making use of the input-output relation of the cavity [34], we have the output field,

$$\begin{aligned}\varepsilon_{out} &= \varepsilon_{in} - 2\kappa c \\ &= \varepsilon_l + \varepsilon_p e^{-i\delta t} - 2\kappa(c_s + c_+ \varepsilon_p e^{-i\delta t} + c_- \varepsilon_p^* e^{i\delta t}),\end{aligned}\quad (7)$$

and thereby the transmission of the probe field is given by [23]

$$t_p = \frac{\varepsilon_p - 2\kappa \varepsilon_p c_+}{\varepsilon_p} = 1 - 2\kappa c_+, \quad (8)$$

which can be measured by the homodyne technique [34].

Defining $\varepsilon_T = 2\kappa c_+$, we obtain the quadrature ε_T of the optical components with frequency ω_p in the output field,

$$\varepsilon_T = 2\kappa \frac{(\delta^2 - \omega_m^2 + i\gamma_m \delta)[\kappa - i(\Delta + \delta)] - 2i\omega_m \beta}{[\Delta^2 + (\kappa - i\delta)^2](\delta^2 - \omega_m^2 + i\delta\gamma_m) + 4\Delta\omega_m \beta}, \quad (9)$$

where $Re[\varepsilon_T]$ and $Im[\varepsilon_T]$ represent the absorptive and dispersive behavior of the optomechanical system, respectively [22].

In order to reduce Eq. (9) and understand the relationship between the charge number and OMIT, we assume and use the following conditions [22, 23]: (i) $\Delta \simeq \omega_m$ and (ii) $\omega_m \gg \kappa$. The first condition means the frequency of the cavity to be in resonance with that of the optomechanical anti-Stokes sideband, which actually leads to optimal cooling. The second condition is the well-known resolved sideband condition, which ensures the OMIT splitting to be distinguished [23]. Moreover, it is known that

the coupling between the MR and the cavity is strongest whenever $\delta \simeq \omega_m$ [22], so $\delta^2 - \omega_m^2 \simeq 2\omega_m(\delta - \omega_m)$ is achievable. Under these conditions, we rewrite the output field as

$$\varepsilon_T = \frac{2\kappa}{\kappa - i(\delta - \omega_m) + \frac{\beta}{\frac{\gamma_m}{2} - i(\delta - \omega_m)}}. \quad (10)$$

Compared with the output field in Eq. (7) in Ref. [22], the parameter β in our case is modified to be a function of the charge number n . Under the choice of appropriate conditions, the window width of the OMIT can be used to identify the charge number of the charged body.

IV. THE CHARGE NUMBER AND THE OUTPUT FIELD

We show below in details how the charge number impacts the mean photon number and how to measure the charge number with the window width of the OMIT.

Using Eq. (5), we have a third-order nonlinear equation for MR position q_s ,

$$aq_s^3 + bq_s^2 + fq_s + d = 0, \quad (11)$$

with

$$\begin{aligned}a &= m\omega_m^2 \frac{\chi^2}{\hbar^2}, \\ b &= -2m\omega_m^2 \frac{\chi}{\hbar} (\Delta_c) - n\eta \frac{\chi^2}{\hbar^2}, \\ f &= m\omega_m^2 \kappa^2 + m\omega_m^2 (\Delta_c)^2 + 2n\eta (\Delta_c) \frac{\chi}{\hbar}, \\ d &= -n\eta \kappa^2 - n\eta (\Delta_c)^2 - \chi |\varepsilon_l|^2.\end{aligned}\quad (12)$$

To get a more intuitive understanding of the role that the Coulomb interaction plays in Eq. (9) and Eq. (10), we suppose $n\eta \gg \chi |c_s|^2$, and obtain the solutions to Eq. (11) as,

$$q_s = \begin{cases} \chi |c_{s0}|^2 / m\omega_m^2, & (n = 0) \\ n\eta / m\omega_m^2, & (n \geq 1) \end{cases} \quad (13)$$

with $|c_{s0}|^2$ being the mean photon number in the absence of the Coulomb interaction between the MR and the charged object. The above equation means that, for no charge in the system, the MR has a steady-state position $q_s = \frac{\chi |c_{s0}|^2}{m\omega_m^2}$ under the action of the pumping field.

However, with opposite charges introduced into the object and the MR, the attractive Coulomb interaction between the charged body and the MR will further modify the steady-state position of the MR, and the steady-state position of the MR is sensitive to this Coulomb force. In the case of $n\eta \gg \chi |c_s|^2$, the steady-state position $q_s = n\eta / m\omega_m^2$ can be reduced to a function of

charge number in the object. These phenomena have been shown in Fig. 2(a).

Moreover, from Eq. (5) and Eq. (13), the mean photon number takes the form of

$$|c_s|^2 = \begin{cases} |c_{s0}|^2, & (n = 0) \\ \frac{|\varepsilon_l|^2}{\kappa^2 + (\Delta_c - \frac{\chi}{\hbar} \frac{n\eta}{m\omega_m^2})^2}, & (n \geq 1) \end{cases} \quad (14)$$

which implies that the photon number increases (decreases) with the charge number for $\Delta_c \geq \frac{\chi}{\hbar} q_s$ ($\Delta_c < \frac{\chi}{\hbar} q_s$) in the case of the fixed pumping field. So there should be a maximal photon number with respect to the change of the charge number, as demonstrated in Fig. 2(b).

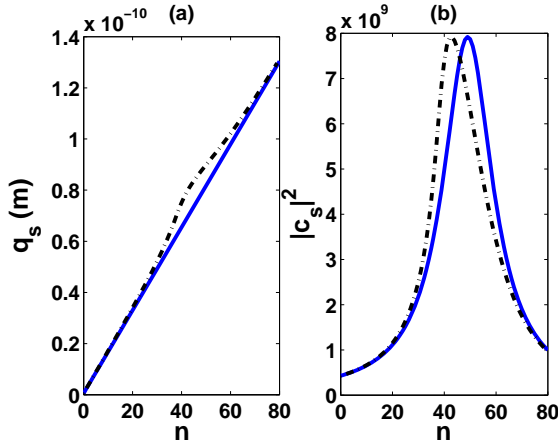


FIG. 2: (Color online) (a) The position q_s versus the charge number n ; (b) The mean photon number as a function of the charge number n . The black dotted-dashed lines [blue solid lines] represent the exact values [the approximate values] of q_s and $|c_s|^2$ using Eq. (11) and Eq. (5) [Eq. (13) and Eq. (14)], respectively. The values are taken from Refs. [4, 5, 22, 35–37] as $\lambda_c \equiv 2\pi c/\omega_c = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\kappa = 2\pi \times 215$ kHz, $\omega_m = 2\pi \times 947$ kHz, $\gamma_m = 2\pi \times 141$ Hz, $r_0 = 67$ μ m, $\varepsilon_l = \sqrt{2P\kappa/\hbar\omega_c}$ with $P = 1$ mW and $Q_{MR} = CU$ with $C = 27.5$ nF and $U = 1$ V.

In Fig. 2, the black dotted-dashed and blue solid lines correspond to the steady-state position and the mean photon number without and with the approximate condition $n\eta \gg \chi|c_s|^2$, respectively. Both the steady-state position and the mean photon number are functions of the charge number, where the former increases monotonously with the charge number, but the latter is a pulse-like curve. Although the slight difference between the exact and approximate values implies our assumption to be reasonable, the deviation of the approximate values from the exact ones should be seriously considered for specific treatment. For the charge number from 30 to 55, the exact values of q_s are larger than the approximate ones, but this is not true for $|c_s|^2$ which reaches the maximum $|c_s|^2 = 7.915 \times 10^9$ at $n = 43$ in the exact treatment

and the maximum $|c_s|^2 = 7.919 \times 10^9$ at $n = 49$ in the approximate one. The deviation results from the fact that the approximate condition $n\eta \gg \chi|c_s|^2$ is not satisfied well with the mean photon number $|c_s|^2 \simeq 0.3n\eta/\chi$. Within the region of $n \leq 40$, both the mean photon number and the MR deformation are ultra-sensitive to the charge number in a monotonous way, and thereby the charge number less than 40 can be fully characterized by the window width of OMIT.

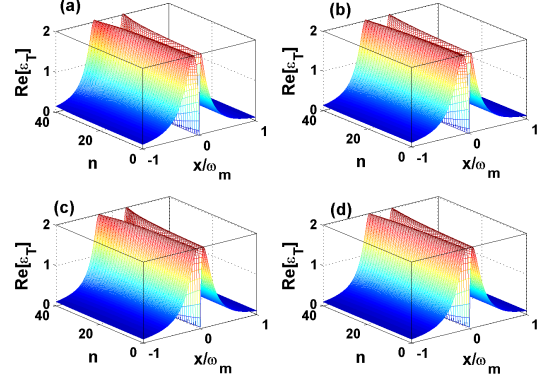


FIG. 3: (Color online) The real part of output $Re[\varepsilon_{out}]$ (the absorption) versus the charge number n and the detuning $x = \delta - \omega_m$, where (a) and (c) are the exact values calculated from Eq. (9). (b) and (d) are the approximate values using Eq. (10). The parameter values are the same as in Fig. 2.

To show this point more clearly, we have simulated the real part of the output field using Eq. (9) and Eq. (10) for $n \leq 40$ (See Fig. 3). Comparing with the exact results, the validity of the simplified expression of the output field Eq. (9) is justified. Moreover, from Fig. 3, we see that the absorption vanishes at $x = 0$ (i.e., $\delta = \omega_m$) and the window width of the OMIT increases with the charge number n . So we are able to detect the charge number of a nearby charged object by the OMIT. In addition, in our case with $n = 0$, the values in the figure can return to those for non-charge case in Ref. [22].

For clarifying the efficiency and effect of our scheme, we consider a fixed charge number n in the charged object. There are three tuning points in the real part of the output field versus the detuning $x = \delta - \omega_m$,

$$\begin{cases} x_{\pm} = \pm \sqrt{\frac{\sqrt{2}(2\kappa + \gamma_m)\sqrt{\beta(2\beta + \kappa\gamma_m)} - \gamma_m(2\beta + \kappa\gamma_m)}{4\kappa}} \\ x_0 = 0 \end{cases}, \quad (15)$$

which can be obtained by solving $d\varepsilon_T/dx = 0$. Excluding the trivial case of $x_0 = 0$ and considering the symmetry of x_+ and x_- , we take the tuning point x_+ as an example in Fig. 4 for different masses and charges. Since the parameters used in Fig. 4 satisfy $2\beta \gg \kappa\gamma_m$, we may reduce Eq. (15) to $x_+ \sim \sqrt{\beta}$. Therefore, by changing the MR's mass m and the MR's charge Q_{MR} , we present three typically

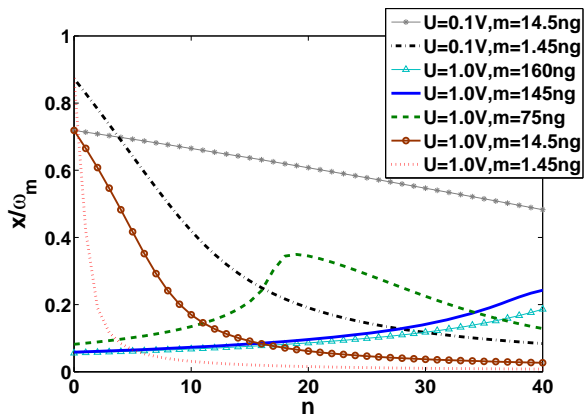


FIG. 4: (Color online) The detuning x for tuning point versus the charge number n . Except the values in the inset, other parameter values are the same as in Fig. 2.

different relationships between the detuning x_+ and the charge number n : (1) monotonous increase (i.e., the gray asterisk and blue solid curves); (2) monotonous decrease (i.e., the red dotted and the black dotted-dashed curves); (3) the curve with a hump in the middle (i.e., the green dashed curve). So the heavy (light) MR is suitable for detecting large (small) charge numbers. Note that, although the MR with intermediate mass is useless in our scheme, the charge number before the hump ($n < 18$) in Fig. 4 can still be used to detect charge number. Moreover, for the same mass of MR, the lower the applied voltage, the less slope the curve in Fig. 4. As a result, to detect the tiny charge, e.g., a single charge, more precisely, we should increase the voltage to obtain more precise resolution.

V. CONCLUSION

We would like to point out that the analytical solutions of our detecting scheme are based on some approxima-

tions ($n\eta \gg \chi|c_s|^2$ and $\delta \sim \omega_m$), which are justified by the parameter values in this work. It has been shown that the MR with an effective mass of 1.45ng and a voltage of 0.1V is most suitable to measure the small charge number (See black dotted-dashed curves in Fig. 4). Experimentally, the effective mass of MR as small as 50pg has been achieved [15]. So we may expect to have better detection with the MRs of such small effective masses.

In addition, the highest sensitivity of the surface charge density in our scheme is about $1/(0.1r_0)^2 \simeq 2.2 \times 10^6 \text{cm}^{-2}$, which is of the same order of magnitude as the one ($6.25 \times 10^6 \text{cm}^{-2}$) in Ref. [38]. But the sensitivity in our case can be further enhanced by increasing the bias gate voltage or decreasing the mass of the MR.

In summary, we have demonstrated how to realize precision measurement of small charge number of the charged object via monitoring the OMIT in optomechanical system in the presence of Coulomb interaction between the charged MR and the charged object. From the analytical relationship we obtained for the window width of OMIT with the charge number in a small charged body, we have shown the possibility of few charges (even single charge) detection from the output spectra of the OMIT. The feasibility of our proposal has been assessed using currently available parameters, and the Coulomb attraction under our consideration can be straightforwardly extended to the Coulomb repulsive case. We believe that the proposal would be helpful for exploring quantum behavior in MRs and for precision measurement using OMIT.

Acknowledgments

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